

## Saigen International College Department of Mathematics and Science Semester 2, 2022

### Year 11 ATAR Mathematics Methods Test 5

(Sequences and series)

Section One (Calculator Free)

Time Allowed: 30 minutes

Total availale marks

Question 1

Given an arithmetic sequence has the terms  $T_{\rm g}=9$  and  $T_{\rm g}=15$  calculate

(9 marks) (6 marks)

the general equation for  $T_{\scriptscriptstyle H}$ 

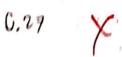
1 = a + (n=1) d

10 = 1.5 + (1.1) \$122 16y.

(ii) the recursive equation for  $\,T_{\scriptscriptstyle B}$ 

To = To-1 11.5, (1)

Express the recurring decimal  $0.\overline{27} = 0.2727...$  as a sum to infinity and hence express it as a (b) rational number (3 marks)

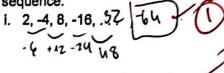


(a) The tenth term of an arithmetic sequence is 98 and the sixteenth term is 80. Determine the sum of the first 20 terms of the sequence. (4 marks)

93 = 
$$\alpha + (9 : -3)$$
  
93 =  $\alpha + 27$   
93 124 :  $\alpha$   
 $\alpha = \frac{80 : \alpha + 16 \times d}{98 - 91} = 80 - 161$   
 $\alpha = \frac{80 : \alpha + 16 \times d}{98 - 91 : \alpha}$   
 $\alpha = \frac{161 + 91}{93}$   
 $\alpha = \frac{20}{2}(2 - 165 + 19 \times -3)$   
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(b) The first two terms of a geometric sequence are  $3 \times 10^{-4}$  and  $6 \times 10^{-6}$ . Calculate the fifth term of the sequence, giving your answer in scientific notation. (4 marks)

(a) State whether the following sequences are Arithmetic or Geometric and state the 6th term of the sequence.



(2 marks)



(2 marks)

- (b) Consider the sequence, -4, 1, 6, ...
  - i. State the recursive definition for the sequence.

(2 marks)

ii. Which term is equal to 56?

(1 marks)

$$66 = -4 + (n-1) \times 5$$

$$60 = (n-1) \times 5$$

$$\frac{60}{5} = n-1$$

$$12 = n-1$$

$$13 = n$$

$$13 + h + erm$$

#### **Question 4**

(5 marks)

The sum of the first n terms of an arithmetic progression are given by

 $S_n = 3n(n-1) - 18n$ 

Determine:

a. T1.

$$S_{n} = 3 \times 4 \times (1 - 1) - 18 \times 1$$

$$S_{1} = 3 \times 0 - 18$$

$$S_{1} = -18$$

$$T_{1} = -18$$

b. The common difference.

(2 marks)

(1 mark)

$$5_2 = 3 \times 2 \times 1 - 18 \times 2$$
  
 $5_2 = 6 - 36$   
 $5_2 = -30 \times -30 + 18 = -12 \times .$   
 $T_2 = ?$ 

(2 marks)

c. Determine n if 
$$S_n = 0$$
.

Over

$$0 = 3n(n-1) - 18n$$

$$0 = 3n^{2} - 3n - 18n$$

$$0 = 3n^{2} - 21n$$

$$0 = 3(n^{2} - 2)$$

$$0 = 3$$

Question 5 (4 marks)

A geometric sequence is described by the rule  $Tn = 5x3^n$ , where  $n = 1, 2, 3, 4, \ldots$  Find the first three terms of the sequence. Hence state the recursive rule for this sequence.

$$T_{n} = 5 \times 3^{n}$$
 $T_{n} = 5 \times 3^{n}$ 
 $T_{n} = 15 \times 3^{n}$ 
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 $T_{n$ 

End of Section One



# Saigon International College Department of Mathematics and Science Semester 2, 2022

### Year 11 ATAR Mathematics Methods Test 5

(Sequence and series)

(6% weighting for the Unit 1 and Unit 2)

Section Two (Calculator Assumed)

39

Time Allowed: 55 minutes

Total available mark: 52

Name of student: . L.W. Mile Day

**Question 6** 

(6 marks)

(a) In a given arithmetic sequence  $T_1 = 7$  and  $T_{20} = 45$ . Evaluate  $S_{20}$ 

(3 marks)

(b) In a given geometric sequence  $T_1 = 256$  and  $T_9 = 1$ . Evaluate  $S_{10}$ 

(3 marks)

$$S_{40} = 256 \times (1 - 0.5^{40})$$

$$-256 \times (1 - 0.5^{40}$$

The table shows the compound growth of an initial investment of \$5000 at the end of each

	+, 2013	E000	$\bigcirc$
3 5 S	End of year	Principal (\$)	Annual Interest (\$)
	2018	5300.00	300.00
	2019	5618.00	318.00
	2020	5955.08	337.08
	2021	6312.38	357.30
	2022	6691.13	378.74
G	23		

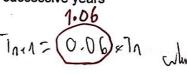
Given  $T_1$  is the start of 2018, state the recursive formula for

(4 marks)

(i) the value of the principal at the end of successive years 0 26 9 UZ 10 28 A A 29 12 30 42

Tn+1 = 4.06 × Tn D Wa T1 =

(ii) the annual interest earned at the end of successive years



(b) calculate the value of the principal at the end of 2030

(2 marks)

### **Question 8**

(10 marks)

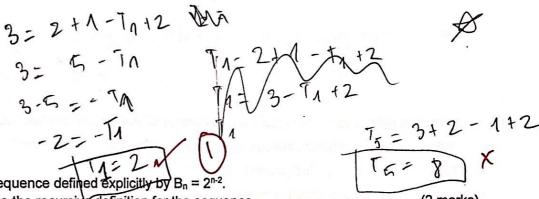
a. A sequence is defined by  $A_{n+1} = 3^n - 2$  where  $A_1 = -1$ . Determine  $A_2$  and  $A_3$ . (2 marks)

$$A_{2} = 3^{2} - 2 \qquad A_{3} = 3^{3} - 2$$

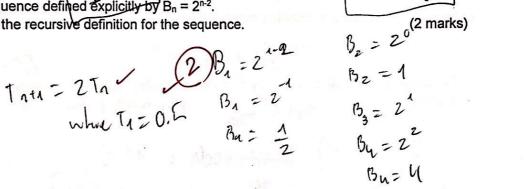
$$A_{2} = 9 - 2 \qquad A_{3} = 27 - 2$$

$$A_{2} = 7 \qquad X \qquad A_{3} = 25$$

 $T_{n+3} = T_{n+2} + T_{n+1} - T_n + 2$  produces a sequence where  $T_2 = 1$ ,  $T_3 = 2$ ,  $T_4 = 3$ . b. (2 marks) Determine T<sub>1</sub> and T<sub>5</sub>.

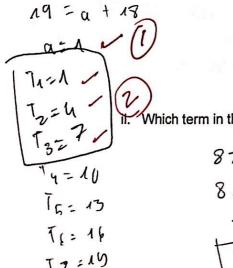


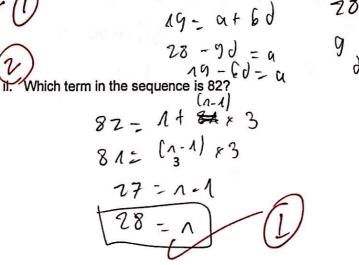
b. A sequence defined explicitly by  $B_n = 2^{n-2}$ Write the recursive definition for the sequence.



- c. The 10th term of an arithmetic progression is 28 and the 7th term is 19.
  - i. Calculate the first three terms of the progression.

(3 marks)



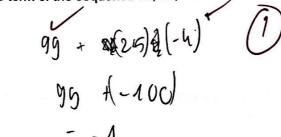


28 = a + [9] )

4-4

(a) Determine the first negative term of the sequence 99, 95, 91.

(3 marks).



n-15 25

first negative term: [26]

(b) A couple gave gifts to various charities over a number of years. Over the last twelve year period they gave gifts annually, increasing in the form of an arithmetic sequence, starting with \$200 twelve years ago, increasing to \$1201 in the twelfth year.

Determine how much they donated over the twelve years.

(4 marks)

$$0 = 200$$
 $1201 = 2001 = 2001 = 110$ 
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### **Question 10**

(10 marks)

a. Determine the sum of the following series.

(2 marks)

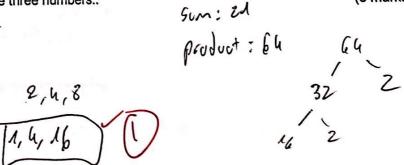


$$\frac{-9^{-3}}{5n^{\frac{1}{2}}} \frac{10^{\frac{2}{3}} \cdot 9^{\frac{5}{3}}}{10^{\frac{2}{3}}} \frac{10^{\frac{5}{3}} \cdot 9^{\frac{5}{3}}}{10^{\frac{5}{3}}} \frac{10^{\frac{5}{3}}}{10^{\frac{5}{3}}} \frac{10^{\frac{5}{3}}}$$

b. How many terms of the series -3 +5 +13 + . . . must be added to give a sum of 261? .(2 marks)



c. Three numbers form a geometric sequence. Their sum is 21 and their product is 64. Determine the three numbers..



d. In a converging geometric series  $S_{\infty} = \frac{3}{2}$  and the sum of the first 3 terms is  $\frac{14}{9}$ . (3 marks) Determine the value of r, the constant ratio.

> Mary = 3  $5 \frac{1}{y} = \frac{(1-1^3)}{1-1}$

Jenny has 6 weeks to train from the City to Surf Marathon.

a. Jenny's training schedule demands that she runs a total of 8000 km. Each week she plans to run a constant number of kilometers further then the week before. If she starts by running 100 km in the first week, how much further she run each week in irder to complete 8oo km planned in the schedule? (3 marks)

a - 100

493.3 km nove each week

- b. Jenny decides she can also increase her fitness level by skipping. She starts with 60 skips a minute and wants to increase the rate by 5% each week.
  - i. How many skips per minutes is she skipping during the last week before the Marathon? (i.e Week 6) (1 mark)

a: 60 /

ii. How many weeks would it take Jenny to be able to skip at over double her initial rate?

during 152 mells

Tor carly # veck 18

\$1,000,000 is invested in an account that pays interest at a rate of 5% per annum compounded annually. Let  $B_n$  be the account balance at the end of n years/.

a. Find the general rule for the account balance at the end of n years. (2 marks)

Find the growth in the account balance in the first 10 years. Hence, find the average percentage growth rate in the first 10 years.

c. Calculate the average percentage growth rate in the first 20 years. (2 marks)

d. Give an explanation for the different answers in the parts((b) and (c). (2 marks)