Saigon International College

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Saigon International College
Department of Mathematics and Science
Semester 2, 2022
Year 11 ATAR Mathematics Methods
Test 5
(Sequences and series)

Section One (Calculator Free)

16

Time Allowed: 30 minutes **Total availale mark: 33**

Student's Name: Chu Linh Dũng

Question 1 **(9 marks)**

(a) Given an arithmetic sequence has the terms $T_5 = 9$ and $T_9 = 15$ calculate (6 marks)

(i) the general equation for T_n

4 step

$15 - 9 = 6 \div 4 = 1.5$

~~$T_n = T_{n-1} + 1.5$~~

$T_n = a + (n-1)d$

$T_n = 1.5 + (n-1) \times 1.5$ ✓ 1

simplify.

(ii) the recursive equation for T_n

$T_n = T_{n-1} + 1.5$, ✓ 1

where $T_1 = 1.5$

(b) Express the recurring decimal $0.\overline{27} = 0.2727...$ as a sum to infinity and hence express it as a rational number (3 marks)

0.27 ✗

Questio

Question 2 **(8 marks)**

(a) The tenth term of an arithmetic sequence is 98 and the sixteenth term is 80. Determine the sum of the first 20 terms of the sequence. (4 marks)

$98 = a + (9 \times -3)$

$98 = a - 27$

$98 + 27 = a$

$\boxed{a = 125}$ 1

$98 = a + 9 \times d$

$80 = a + 15 \times d$

$98 - 9d = a$

$80 - 15d = a$

$98 - 9d = 80 - 15d$

$18 = -15d + 9d$

$18 = -6d$

$d = -3$

$S_{20} = \frac{20}{2}(2 \times 125 + 19 \times -3)$

$S_{20} = 10 \times (250 - 57) = 10 \times 193 = \boxed{1930}$ 1 1

2
19
$\times 3$
57

(b) The first two terms of a geometric sequence are 3×10^{-4} and 6×10^{-6}. Calculate the fifth term of the sequence, giving your answer in scientific notation. (4 marks)

0.0003.

0.000006

~~0.000006~~

0.

$6 \div 300 =$

0.0003×0.02^5

$300 \overline{)6}$ = 0.02

$0.0003 \times \cancel{0.02}$

0.0003×0.0000032

$0.02 \times 0.02 = 0.004$

$\times 0.02 = 0.0008$

$\times 0.02 = 0.000016$

0.0000032

$\times 3 = 96$

0.00000000096

$\boxed{9.6 \times 10^{-9}}$

X

Question 3 **(7 marks)**

(a) State whether the following sequences are Arithmetic or Geometric and state the 6^{th} term of the sequence.

i. 2, -4, 8, -16, . 32 $\boxed{-64}$ 1 **(2 marks)**

-4 +12 -24 48

geometric 1

1 2 3

ii. a, ax^2, ax^4, . . ax^6 ax^8 $\boxed{ax^{10}}$ **(2 marks)**

arithmetic

+5 +5

(b) Consider the sequence, -4, 1, 6, . . .

i. State the recursive definition for the sequence. **(2 marks)**

$T_n = T_{n-1} + 5$ 1

where $T_1 = -4$ 1

ii. Which term is equal to 56? **(1 marks)**

$56 = -4 + (n-1) \times 5$

$60 = (n-1) \times 5$

$\frac{60}{5} = n-1$

$12 = n-1$

$\boxed{13 = n}$ 1 13th term

Question 4 **(5 marks)**

The sum of the first n terms of an arithmetic progression are given by

$S_n = 3n(n-1) - 18n$

Determine:

a. T1. (1 mark)

$S_n = 3 \times 1 \times (1-1) - 18 \times 1$

$S_1 = 3 \times 0 - 18$

$S_1 = -18$

$\boxed{T_1 = -18}$ ✓ 1

b. The common difference. (2 marks)

$S_2 = 3 \times 2 \times 1 - 18 \times 2$

$S_2 = 6 - 36$

$S_2 = -30$ ✓ $-30 + 18 = \boxed{-12}$ x.

$T_2 = ?$.

c. Determine n if $S_n = 0$. (2 marks)

$\boxed{T_{-0.5}}$

~~3×1~~

$0 = 3n(n-1) - 18n$ ✓ 1

$0 = 3n^2 - 3n - 18n$

$0 = 3n^2 - 21n$

$0 = \cancel{3(n^2 - 7n)}$

$0 = 3n(n-7)$

$0 = n - 7$?

$n = 7$

Question 5 **(4 marks)**

A geometric sequence is described by the rule $Tn = 5x3^n$, where n = 1, 2, 3, 4, ... Find the first three terms of the sequence. Hence state the recursive rule for this sequence.

$T_n = 5 \times 3^n$

$T_1 = 5 \times 3^1$

$T_1 = 15$ ✓ 1

$T_2 = 5 \times 3^2$

$T_2 = 5 \times 9$

$T_2 = 45$ ✓ 1

$T_3 = 5 \times 3^3$

$T_3 = 5 \times 27$

$T_3 = 135$ ✓ 1

$$\begin{array}{r} \overset{3}{27} \\ \times \; 5 \\ \hline 135 \end{array}$$

$\boxed{15, 45, 135}$

End of Section One

~~$T_n = T_{n-1} \times$~~

$T_{n+1} = 3T_n$ ✓ 1

where $T_1 = 15$

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Semester 2, 2022
Year 11 ATAR Mathematics Methods
Test 5
(Sequence and series)
(6% weighting for the Unit 1 and Unit 2)
Section Two (Calculator Assumed)

39

Time Allowed: 55 minutes **Total available mark: 52**

Name of student : Chu Minh Đăng

Question 6 **(6 marks)**

(a) In a given arithmetic sequence $T_1 = 7$ and $T_{20} = 45$. Evaluate S_{20} (3 marks)

$a = 7$

$45 = 7 + 19 \times d$

$45 - 7 = 19d$

$38 = 19d$

$d = 2$

$S_{20} = 10 \times (2 \times 7 + 19 \times 2)$

$S_{20} = 10 \times (14 + 38)$

$S_{20} = 10 \times 52$

$\boxed{S_{20} = 520}$

3

(b) In a given geometric sequence $T_1 = 256$ and $T_9 = 1$. Evaluate S_{10} (3 marks)

$1 = 256 + 8 \times d$

$-255 = 8d$

$d = \frac{-255}{8}$

$T_1 = ar^{n-1}$

$1 = 256 \times r^8$

$r = 0.5$

1

$S_{10} = \frac{256 \times (1 - 0.5^{10})}{1 - 0.5}$

1

$\boxed{S_{10} = 511.5}$

1

Question

Question 7 **(6 marks)**

The table shows the compound growth of an initial investment of $5000 at the end of each successive year.

t_1 2018 5000 0

End of year	Principal ($)	Annual Interest ($)
2018	5300.00	300.00
2019	5618.00	318.00
2020	5955.08	337.08
2021	6312.38	357.30
2022	6691.13	378.74

0.06

0.06

3 4 5 6 7 8 9 10 11 12 13

23 25 26 27 28 29 30

(a) Given T_1 is the start of 2018, state the recursive formula for **(4 marks)**

(i) the value of the principal at the end of successive years

$T_{n+1} = 1.06 \times T_n$ 1

where $T_1 = 5300$ 1

(ii) the annual interest earned at the end of successive years

$T_{n+1} = 0.06 \times T_n$ 1.06 1

where $T_1 = 300$

(b) calculate the value of the principal at the end of 2030 **(2 marks)**

2030 - 2018 = 12

$T_{13} = 5300 \times 1.06^{12}$

$\boxed{T_{13} = \$10\,664.64}$ 2

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Question 8 **(10 marks)**

a. A sequence is defined by $A_{n+1} = 3^n - 2$ where $A_1 = -1$. Determine A_2 and A_3. (2 marks)

$A_2 = 3^2 - 2$ $\quad A_3 = 3^3 - 2$

$A_2 = 9 - 2$ $\quad A_3 = 27 - 2$

$A_2 = 7$ X $\quad A_3 = 25$

b. $T_{n+3} = T_{n+2} + T_{n+1} - T_n + 2$ produces a sequence where $T_2 = 1$, $T_3 = 2$, $T_4 = 3$. Determine T_1 and T_5. (2 marks)

$3 = 2 + 1 - T_1 + 2$

$3 = 5 - T_1$

$3 - 5 = -T_1$

$-2 = -T_1$

$\boxed{T_1 = 2}$ ✓ 1

$T_5 = 3 + 2 - 1 + 2$

$\boxed{T_5 = 8}$ X

b. A sequence defined explicitly by $B_n = 2^{n-2}$.
Write the recursive definition for the sequence. (2 marks)

$T_{n+1} = 2T_n$ ✓

where $T_1 = 0.5$ ✓ 2

$B_1 = 2^{1-2}$

$B_1 = 2^{-1}$

$B_n = \frac{1}{2}$

$B_2 = 2^0$

$B_2 = 1$

$B_3 = 2^1$

$B_4 = 2^2$

$B_n = 4$

c. The 10th term of an arithmetic progression is 28 and the 7th term is 19.

i. Calculate the first three terms of the progression. (3 marks)

$28 = a + (9)d$

$19 = a + 6d$

$28 - 9d = a$

$19 - 6d = a$

$28 - 9d = 19 - 6d$

$9 = 3d$

$d = 3$ ✓

$19 = a + 18$

$a = 1$ ✓ 1

$T_1 = 1$ ✓

$T_2 = 4$ ✓ 2

$T_3 = 7$ ✓

$T_4 = 10$

$T_5 = 13$

$T_6 = 16$

$T_7 = 19$

ii. Which term in the sequence is 82? (1 mark)

$82 = 1 + (n-1) \times 3$

$81 = (n-1) \times 3$

$27 = n - 1$

$\boxed{28 = n}$ ✓ 1

Question 9 **(7 marks)**

$-4 \quad -4$

(a) Determine the first negative term of the sequence 99, 95, 91. (3 marks).

$99 + (25)(-4)$ 1

$99 + (-100)$

$= -1$

$n - 1 = 25$

$n = 26$

first negative term: $\boxed{26}$ 1 1

(b) A couple gave gifts to various charities over a number of years. Over the last twelve year period they gave gifts annually, increasing in the form of an arithmetic sequence, starting with $200 twelve years ago, increasing to $1201 in the twelfth year.

Determine how much they donated over the twelve years. (4 marks)

$a = 200$ 1

$1201 = 200 + (11)d$

$1001 = 11d$

$d = 91$ 1

$S_{12} = \frac{12}{2}(2 \times 200 + 11 \times 91)$ 1

$S_{12} = 6(400 + 1001)$

$S_{12} = \$8406$ 1

Question 10 **(10 marks)**

a. Determine the sum of the following series. (2 marks)

35+ 32 + 29 + . . . + 5

-3 -3

a = 35

d = -3

10 = n-1

11 = n

$S_{11} = \frac{11}{2}[2 \times 35 + 10 \times -3]$

$\cancel{S_{10}} = T_{10} = 35 + 10 \times -3$

$T_{10} = 5$

$\boxed{S_{11} = 220}$

2

b. How many terms of the series -3 +5 +13 + . . . must be added to give a sum of 261? .(2 marks)

d = +8 1

a = -3

$261 = \frac{n}{2}[a$

$\boxed{9 \text{ terms}}$ 1

c. Three numbers form a geometric sequence. Their sum is 21 and their product is 64. Determine the three numbers.. (3 marks)

sum: 21

product: 64

64, 32, 2, 16, 2

2, 4, 8

$\boxed{1, 4, 16}$ 1

d. In a converging geometric series $S_\infty = \frac{3}{2}$ and the sum of the first 3 terms is $\frac{14}{9}$.

Determine the value of r, the constant ratio. (3 marks)

$\frac{a}{1-r} = \frac{3}{2}$ 1

$\frac{14}{9}$ $\cancel{S_3} = \frac{a(1-r^3)}{1-r}$

5

$\boxed{r = -0.3\bar{3}}$ 1

$a = \frac{3}{2}(1-r)$ 1

Question 11 **(5 marks)**

Jenny has 6 weeks to train from the City to Surf Marathon.

a. Jenny's training schedule demands that she runs a total of 8000 km. Each week she plans to run a constant number of kilometers further then the week before. If she starts by running 100 km in the first week, how much further she run each week in irder to complete 8000km planned in the schedule? (3 marks)

arith

$a = 100$

493.3 km more each week

1

b. Jenny decides she can also increase her fitness level by skipping. She starts with 60 skips a minute and wants to increase the rate by 5% each week.

i. How many skips per minutes is she skipping during the last week before the Marathon? (i.e Week 6) (1 mark)

$a = 60$

$r = 1.05$

~~80.4 skips per minute~~

1

76.6 skips per minute

ii. How many weeks would it take Jenny to be able to skip at over double her initial rate? (1 mark)

week 15 1

during 15.2 weeks

or early ~~15~~ week 16

$14.2 = x - 1$

$15.2 = x$

Question 12 **(8 marks)**

$1,000,000is invested in an account that pays interest at a rate of 5% per annum compounded annually. Let B_n be the account balance at the end of n years/. 1.05

a. Find the general rule for the account balance at the end of n years. (2 marks)

$B_n = 1000000 \times 1.05^{n-1}$ n 1

b. Find the growth in the account balance in the first 10 years. Hence, find the average percentage growth rate in the first 10 years. (2 marks)

1551328.216

~~55.1~~ 5.5%/year X

c. Calculate the average percentage growth rate in the first 20 years. (2 marks)

0.0763

7.63%/year X

d. Give an explanation for the different answers in the parts((b) and (c). (2 marks)

Because the growth rate is compounded annually so at later times, the interest earned is a bit greater than the interest ~~when~~ at earlier years

2

End of section 2